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Pearson Edexcel
Level 3 Advanced GCE in Mathematics (9MA0)

May/June 2023 Exemplar
9MA0-32 A level Mathematics
Paper 32: Mechanics
Senior Examiner's feedback on student responses

Contents

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About this booklet

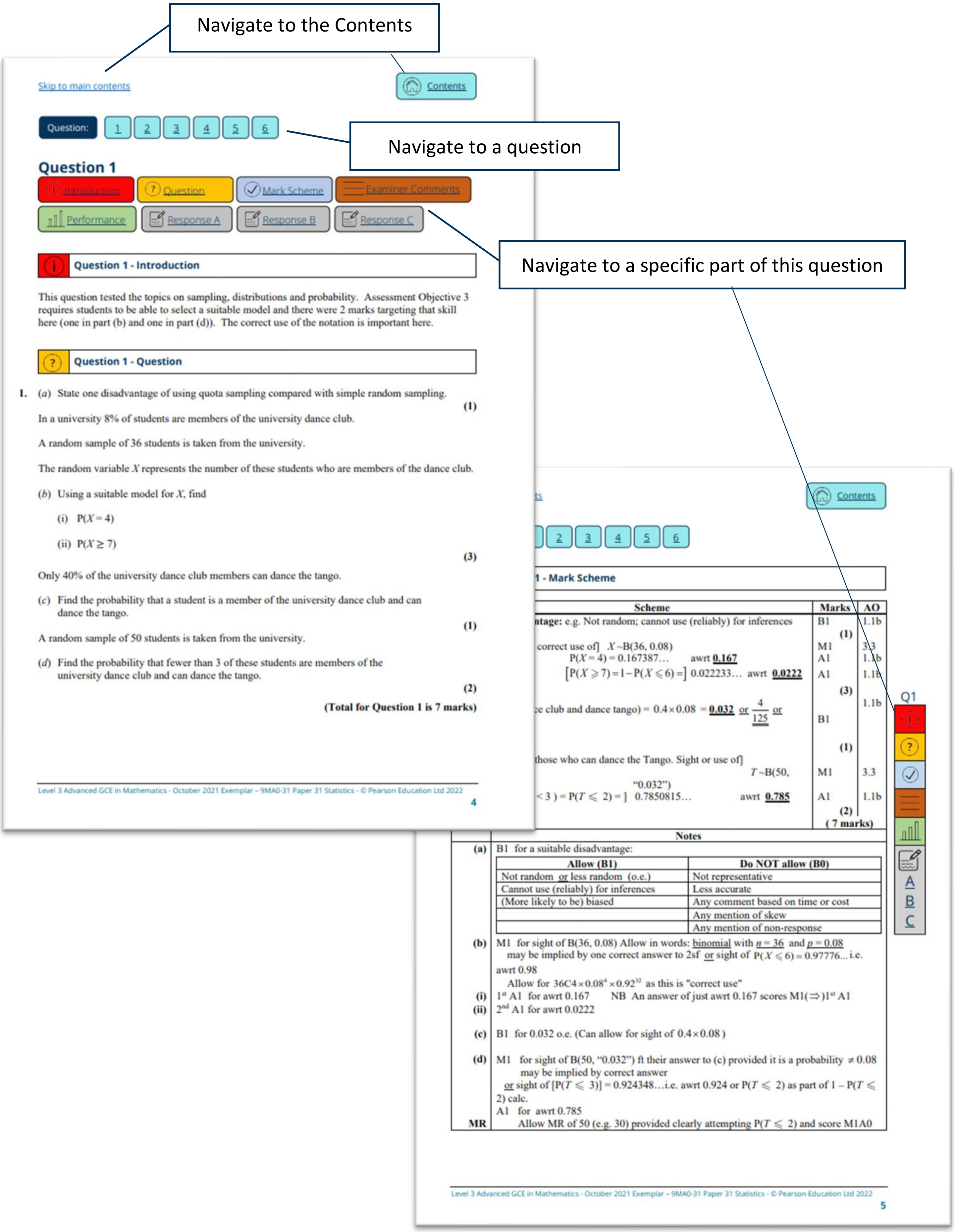
This document has been produced to support mathematics teachers delivering the GCE Mathematics specification.

This document looks at questions from the 9MA0-32 A level Mathematics Paper 32: Mechanics June 2023 examination paper. It shows real student responses to these questions, and how the examining team follow the mark schemes to demonstrate how the students are awarded the marks. For the mark scheme notes and details of alternative methods please see the full mark scheme for this question paper on our website.

For 2023, the approach all exam boards have taken to grading was to return to pre-pandemic grading, while giving students protection against any impact of disruption. Results in summer 2023 therefore will be far more in line with summer exams that were sat in 2019, but lower than in 2022, when grades awarded were based on a mid-point between 2019 and 2021 outcomes. For more information please read our 'Understanding grade boundaries 2023' document.

* The question level performance data is there to give an indication only of how students performed, on each question, in the context of sitting the entire exam paper and is not an indication of how students may perform sitting a question in isolation.

How to use this booklet



General Examiner Feedback

The paper seemed to work particularly well with the majority of candidates able to make attempts at all six questions, although there was some evidence of time issues for weaker candidates. The first three questions provided the opportunity for candidates to settle into the paper and score some easy marks. The remaining questions ramped nicely in their accessibility. There were some excellent scripts but there were also some where there were issues with the standard of presentation. This, in some cases, made it difficult for examiners to follow the working. Candidates should try to spread their work out as this will make it easier to read.

Question 1 was the best answered question, followed by questions 2 to 5.

The most challenging question was question 6. Questions 4, 5 and 6 produced a similar pattern of responses, with a small proportion unable to make any progress at all but large numbers scoring all or almost all of the marks.

In calculations the numerical value of g which should be used is 9.8. Final answers should then be given to 2 (or 3) significant figures – more accurate answers will be penalised, including fractions but exact multiples of g are usually accepted.

There were a number of printed answers to show on this paper, and candidates *must* ensure that they show sufficient detail in their working to warrant being awarded all of the marks available and that they end up with ***exactly*** what is printed on the question paper. There were many cases where it was similar (e.g. $S = \frac{1}{2}\cot\theta Mg$ in 6(b)) but not *exactly* as printed. Candidates run the risk of losing a mark in such cases.

In all cases, as stated on the front of the question paper, candidates should show sufficient working to make their methods clear to the examiner and correct answers without working may not score all, or indeed, any of the marks available.

If a candidate runs out of space in which to give their answer than they are advised to use a supplementary sheet – if a centre is reluctant to supply extra paper then it is crucial for the candidate to say whereabouts in the script the extra working is going to be done.

Question 1

Question 1 - Introduction

This question tests a candidate's ability to apply the *suvat* formulae in a simple scenario.

Question 1 - Question

1. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of 3.2 m s^{-2}

Find

(*a*) the speed of the car after 5 s,

(1)

(*b*) the distance travelled by the car in the first 5 s.

(2)

(Total for Question 1 is 3 marks)

Question 1 - Mark Scheme

Question	Scheme	Marks	AOs
1(a)	16 (m s^{-1}) seen as the answer	B1	1.1b
		(1)	
1(b)	$s=\frac{1}{2}\times 3.2\times 5^2$ **OR** $s=\frac{(0+16)}{2}\times 5$ **OR** $s=(16\times 5)-\frac{1}{2}\times 3.2\times 5^2$ **OR** $16^2=2\times 3.2\times s$ **OR** from a *v*-*t* graph, $s=\frac{1}{2}\times 5\times 16$	M1	3.1b
	s = 40 (m)	A1	1.1b
		(2)	
			(3 marks)

Notes:

1a	B1	cao. Must be positive. Ignore any working.
1b	M1	Complete method to find an equation in s only, possibly using their '16' Allow 'reversed motion': use of $s=vt-\frac{1}{2}at^2$ with v = 0 i.e. $s=-\frac{1}{2}\times 3.2\times 5^2$ can score M1 and s = –40 so distance is 40 (m) can score the A1
	A1	cao. Must be positive.
		N.B. correct answer only, in (b), can score both marks.

Q1
A
B
C

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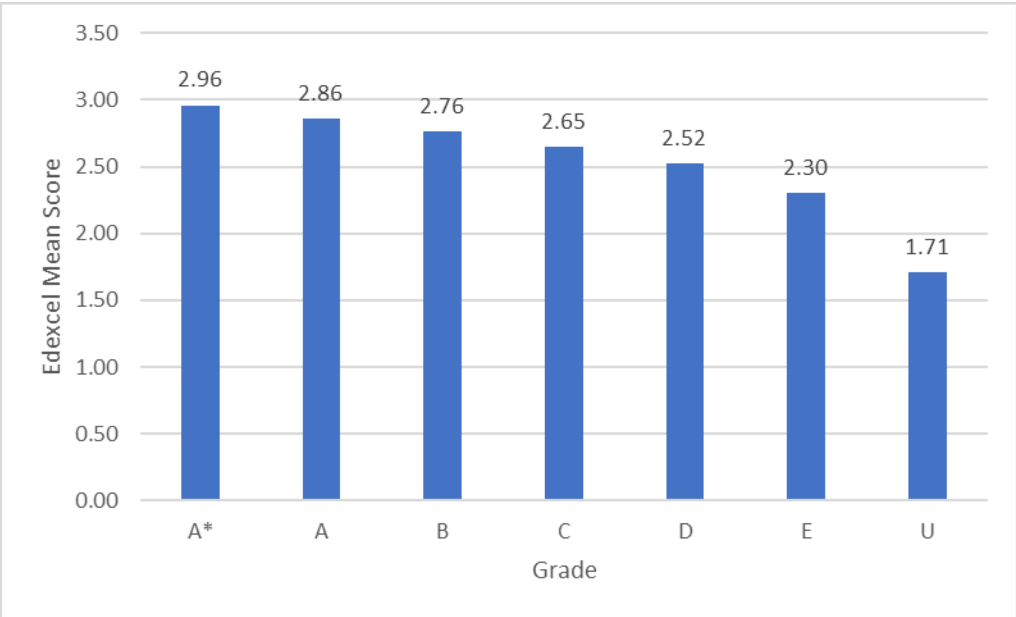
Question: 1 2 3 4 5 6

Question 1 - Examiner Comments

This proved to be a very friendly starter. In part (a), almost all of the candidates were able to score the mark. In some rare instances it was lost for incorrect subsequent working such as $\sqrt{16} = 4$. In the second part, the vast majority used a correct method with most using $s = ut + \frac{1}{2}at^2$ but other *suvat* formulae were seen. Area under a *v-t* graph and integration were employed very occasionally but usually with success. The most common mistake was to assume constant velocity and a few made an error with the arithmetic and lost the final mark. A surprising number wrote 3.2 for acceleration but used 3.5 in their calculation. Candidates would be well advised to state the formula being used before doing their calculation, so that method marks can still be awarded in the event of an arithmetical slip.

Question 1 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
2.69	3	90%	2.96	2.86	2.76	2.65	2.52	2.30	1.71



Q1

A
B
C

Question 1 - Response A

a) $3.2 \times 5 = 16\,ms^{-1}$

b) $\cancel{s = ut + \frac{1}{2}at^2}$ speed = $\frac{\text{distance}}{\text{time}}$

$\cancel{s=}$ distance = speed × time

$16 \times 5 = 80\,m$

1/3 marks

Part (a)
B1: 16 is seen

Part (b)
M0: Uses $s = vt$. This was a common misconception, treating it as constant speed.
A0: After M0

Q1
i
?
A
B
C

Question 1 - Response B

$a = 3.2\,ms^{-2}$

(a). $t = 5,\ a = 3.2,\ u = 0.$

$v = u + at$

$v = 0 + 3.2 \times 5$

$v = 16$

(b) $S = vt - \frac{1}{2}at^2$

$= 16 \times 5 - \frac{1}{2} \times 3.2 \times 5^2$

$\cancel{= 8 \times 5}$

$= 80 - 8$

$= 72\,m.$

2/3 marks

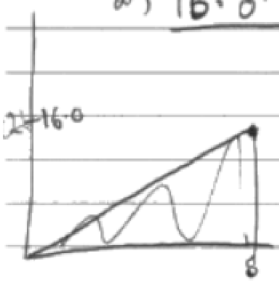
Part (a)
B1: 16 is seen.

Part (b)
M1: Complete method to find *s*, with formula clearly stated - makes a slip forgetting to square 5. The disadvantage of using this method is that it uses the answer to (a) which could be incorrect.
A0: Incorrect answer.

Q1 A B C

Question 1 - Response C

a) 16·0



b) $\frac{16\times5}{2} = \frac{80}{2} = \underline{40}$

3/3 marks

Part (a)
B1: 16 is seen.

Part (b)
M1: Complete method to find the distance using the area under the *v*-*t* graph.
A1: Correct answer.

Q1
A
B
C

Question 2

Introduction | Question | Mark Scheme | Examiner Comments
Performance | Response A | Response B | Response C

Question 2 - Introduction

This question tests a candidate's ability to use Newton's Laws and knowledge of friction in a dynamic situation.

Question 2 - Question

2.

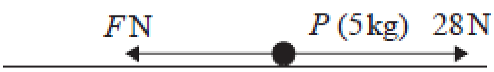


Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at 1.4 m s^{-2}

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

(Total for Question 2 is 4 marks)

Question 2 - Mark Scheme

Question	Scheme	Marks	AOs
2(a)	Resolve vertically, $R = 5g = 49$ (N)	B1	1.1b
		(1)	
2(b)	Equation of motion: $28 - F = 5 \times 1.4$	M1	3.1a
	$F = 21$	A1	1.1b
		(2)	
2(c)	$\mu = 0.43$ (2sf required)	B1 **ft**	3.4
		(1)	
			(4 marks)

Notes:

2a	B1	Allow either $5g$ or 49. No penalty for using $g = 9.81$ or 10. Ignore any working. Must be a positive number. B0 if m is involved. **N.B**. Could be seen on a diagram, provided it's clearly the reaction.
2b	M1	Equation with correct terms, dimensionally correct, condone sign errors.
	A1	cao but allow $\frac{15g}{7}$. Ignore units.
2c	B1**ft**	$\mu = \frac{\text{their (b)}}{\text{their (a)}}$. **Answer must be a positive number given to 2sf**. **N.B.** B0 if they use $g = 9.81$ or 10 in this part of the question. Do not allow restarts. Allow $\mu > 1$.

Q2

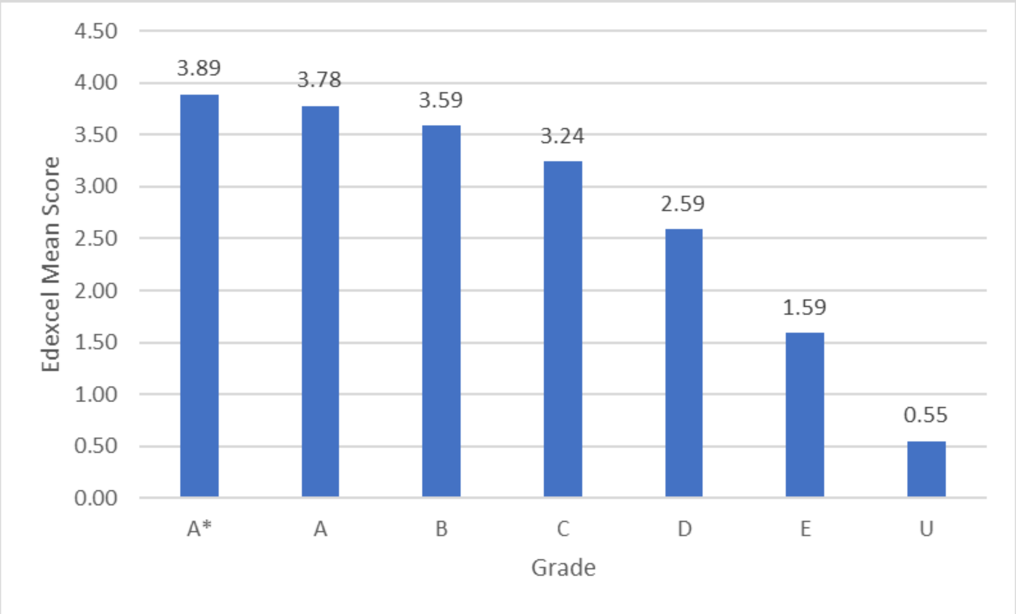
A
B
C

Question 2 - Examiner Comments

The vast majority were able to score the mark in part (a), with answers of $5g$ or 49 being the most common. A few candidates used $g = 9.81$ but were not penalised here. There were very many correct answers for the second part also, but some ignored the 28N force completely and gave $5\times1.4=7$, as their answer or made a sign error and ended up with $F = 35$. In the final part, $F = \mu R$ was very well known, and this was a follow through mark on their answers to (a) and (b). However, some left their answer as a fraction or didn't give it to 2 sf and lost the mark anyway.

Question 2 - Performance

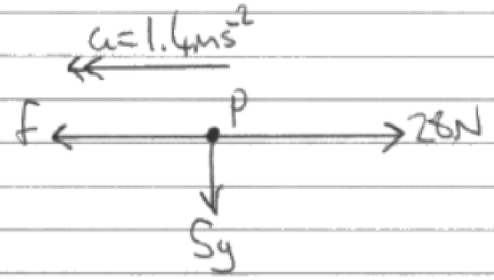
			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
3.24	4	81%	3.89	3.78	3.59	3.24	2.59	1.59	0.55



Question 2 - Response A

a) Normal Reaction, $R = 5 \times 28$
$R = 140\,N$

b)



$F = ma$
$F = 5 \times 1.4$
$F = 7N$

c) $F = \mu R$ $\quad \mu = \frac{7}{140}$
$\mu = \frac{F}{R}$ $\quad \mu = 0.05$

1/4 marks

Part (a)
B0: Incorrect answer.

Part (b)
M0: Incorrect equation of motion - only has the *ma* term - both forces missing.
A0: Following M0.

Part (c)
B1ft: Correct answer (to 2sf although technically only 1sf but this was allowed) using their *F* and *R*.

Question 2 - Response B

Normal reaction force $= 5 \times 9.81 = 49.05\,N$

b) $28 - F = ma$

$28 - F = 5 \times \cancel{[illegible]}\ 1.04$

$28 - 11.4 = F$

$F = 16.6$

c) $\frac{16.6}{49.05} = \mu$

$0.338 = \mu$

2/4 marks

Part (a)
B1: Correct answer. Use of g = 9.81 was not penalised.

Part (b)
M1: Correct equation of motion.
A0: Incorrect answer.

Part (c)
B0ft: Uses their F and R correctly but answer not given to 2sf.

Q2

A
B
C

Question 2 - Response C

a) $5 \times 9.8 = 49$

b). $F = ma$

$5g \times 1.4$

$= 49 \times 1.4 = 68.6$

~~68.6~~ $68.6 - 28 = FN$

$40.6 = FN$

c) N R

0.83 R

$R = 5(9.8)$

$= 49$

$\frac{49}{40.6} =$ ~~1.2070~~

$= 0.83$

3/4 marks

Part (a)
B1: Correct answer.

Part (b)
M1: Equation has a sign error and an extra *g* in the *ma* term; both condoned for the M mark.
A0: Incorrect answer.

Part (c)
B1ft: Correct 2sf answer, ft on their answers to (a) and (b).

Q2

A

B

C

Question 3

Introduction | Question | Mark Scheme | Examiner Comments

Performance | Response A | Response B | Response C

Question 3 - Introduction

This question tests a candidate's knowledge of variable acceleration in a vectors context, using calculus.

Question 3 - Question

3. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}$ m s^{-1} where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(*a*) the speed of P at time $t = 0$

(3)

(*b*) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$

(2)

(*c*) the acceleration of P at time t seconds

(2)

(*d*) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$

(2)

(Total for Question 3 is 9 marks)

Question 3 - Mark Scheme

Question	Scheme	Marks	AOs
3(a)	$7\mathbf{i} - 3\mathbf{j}$ seen or implied by Pythagoras	B1	1.1b
	Use Pythagoras: $\sqrt{7^2 + (-3)^2}$	M1	3.1a
	$\sqrt{58}$, 7.6 or better (m s^{-1})	A1	1.1b
		(3)	
3(b)	$t^2 - 3t + 7 = 2t^2 - 3$ **OR** $\dfrac{t^2 - 3t + 7}{2t^2 - 3} = \dfrac{1}{1} = 1$	M1	2.1
	$t = 2$ only	A1	1.1b
		(2)	
3(c)	Differentiate $\mathbf{v}$ wrt t to give a vector.	M1	3.1a
	$(2t - 3)\mathbf{i} + 4t\mathbf{j}$	A1	1.1b
		(2)	
3(d)	$2t - 3 = 0$	M1	3.1a
	$t = 1.5$	A1	1.1b
		(2)	
		(9 marks)	

Notes: Allow column vectors throughout.

3a	B1	cao
	M1	Use of Pythagoras, including the square root, on a **velocity** vector at $t = 0$
	A1	cao. Must come from a <u>correct</u> **v**.
3b	M1	Equating **i** and **j** components of **v** or a ratio of 1:1 to obtain a quadratic in t only. If they use a constant, e.g. $t^2 - 3t + 7 = k$ and $2t^2 - 3 = k$, k must be eliminated to earn this mark. **N.B.** M0 (since wrong working seen) if they write down $\mathbf{i} + \mathbf{j} = \left(t^2 - 3t + 7\right)\mathbf{i} + \left(2t^2 - 3\right)\mathbf{j}$ **OR** $\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}t^2 - 3t + 7\\2t^2 - 3\end{pmatrix}$ **OR** $t^2 - 3t + 7 = 1$ and $2t^2 - 3 = 1$ and then $t^2 - 3t + 7 = 2t^2 - 3$

Q3

i

?

A

B

C

	A1	$t = 2$
		N.B. Allow M1A1 for a **correct** trial and error method where they obtain $\mathbf{v} = 5\mathbf{i} + 5\mathbf{j}$ when $t = 2$ but M0 if they don't get $t = 2$
3c	M1	At least one power decreasing by 1 in **each** component in their **v** (M0 if clearly dividing by t) **Both i** and **j** needed in their answer or a column vector Allow recovery if the **i** and **j** disappear and then reappear.
	A1	cao (must be a vector) isw e.g. if they find the magnitude or put $t = 0$ or differentiate again **i**'s and **j**'s do not need to be collected. **N.B.** Allow M1A0 for $2t - 3\mathbf{i} + 4t\mathbf{j}$
3d	M1	$2t - 3 = 0$ or (their **derivative** of the **i**-component of **v**) = 0 **N.B.** M0 if they equate the derivative of both components of **v** to zero.
	A1	cao **N.B.** Correct answer, with no working, can score both marks.

Question 3 - Examiner Comments

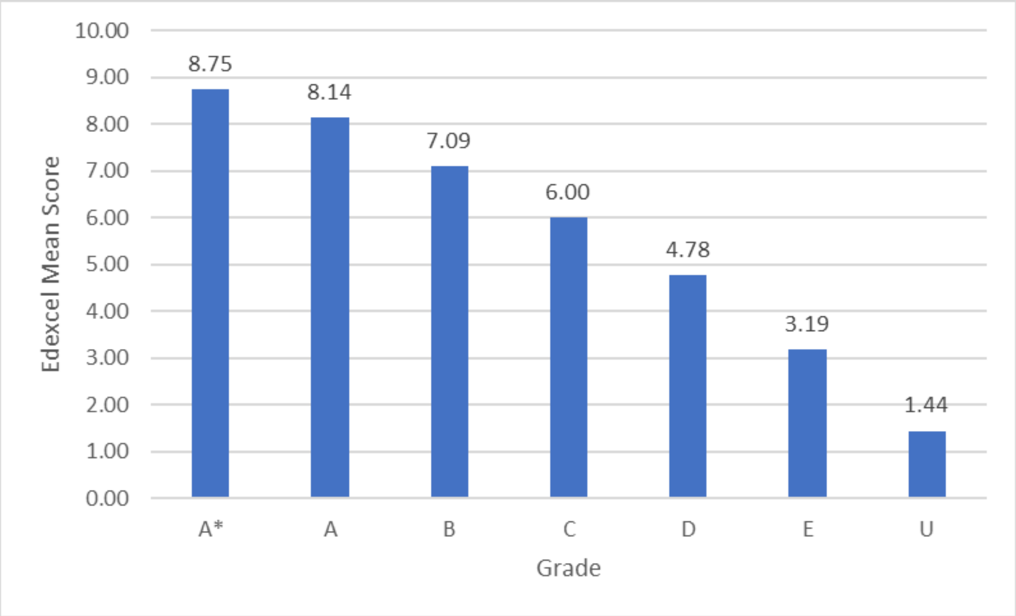
Part (a) was generally a well answered question. Some found the velocity vector at $t = 0$ and went no further, either forgetting, or not being aware that they needed to use Pythagoras' Theorem to find the speed. A small number of candidates scored the M mark only, if they had an incorrect velocity vector but used Pythagoras' Theorem correctly. The majority of candidates who attempted the second part used a correct method, setting the **i** and **j** components of the velocity equal to each other or both equal to a constant which was then eliminated. A few also successfully used trial and error. However, a significant number set both **i** and **j** coefficients equal to 1, solved one or two separate equations and scored nothing.

Q3

A
B
C

Question 3 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade: A*	A	B	C	D	E	U
6.63	9	74%	8.75	8.14	7.09	6.00	4.78	3.19	1.44



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Question: 1 2 3 4 5 6

Question 3 - Response A

(a) at $t=0$

$v=(t^2-3t+7)\underline{i}+(2t^2-3)\underline{j}$

$\underline{v}=(0^2-3(0)+7)\underline{i}+(2(0)^2-3)\underline{j}$

$=7\underline{i}+-3\underline{j}$

(b) $\underline{i}+\underline{j}=(t^2-3t+7)\underline{i}+(2t^2-3)\underline{j}$

$0=(t^2-3t+6)\underline{i}+(2t^2-4)\underline{j}$

$2t^2-4=t^2-3t+6$

$0=-t^2-3t+10$

$t^2+3t-10=0$

$t=2,-5$

-5 cant be t so $t=2$

Q3

A

B

C

(c) $v = (t^2 - 3t + 7)\underline{i} + (2t^2 - 3)\underline{j}$

$v = (4 - 6 + 7)\underline{i} + (2(4) - 3)\underline{j}$

$V = 5\underline{i} + 5\underline{j}$ ms^{-1}

acceleration $= \frac{V}{t}$ where $t = 2$

$\frac{5}{2}\underline{i} + \frac{5}{2}\underline{j}$ ms^{-2}

(d) when $\underline{j} = 0$

$2t^2 - 3 = 0$

$2t^2 = 3$

$t^2 = \frac{3}{2}$

$t = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$ or 1.22 seconds.

1/9 marks

Part (a)
B1: Correct answer seen for the velocity.
M0: No use of Pythagoras.
A0: After M0.

Part (b)
M0: Uses an incorrect method with wrong working seen, uses equations rather than ratios.
A0: After M0.

Part (c)
M0: No attempt to differentiate **v** with respect to *t* (divides by *t*).
A0: Follows M0.

Part (d)
M0: Has no derivative of the **i** component of **v**
A0: Follows M0.

Q3

i

?

A
B
C

Question 3 - Response B

a) $t=0$

$V = 7i - 3j$

$\sqrt{(7i)^2 (-3j)^2} = 2\sqrt{10}$

b) p v a $t^2 - 3t + 7 = 2t^2 - 3$

$3t^2 - 3t + 10$

c) $\cancel{\int (t^2 - 3t + 7)i + (2t^2 - 3)j}$

$\cancel{3t^3}$

$\cancel{a = (\frac{1}{3}t^3 - \frac{3t^2}{2} + 7t)i + (\frac{2}{3}t^2 - 3)}$

c) $(2t - 3)\cancel{4}\underset{\sim}{i} + (4t)\cancel{2}\underset{\sim}{j}$

d) $t = 3$

4/9 marks

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Question: 1 2 3 4 5 6

Part (a)
B1: Correct answer seen.
M0: Incorrect use of Pythagoras. Subtracts squares.
A0: Follows M0.

Part (b)
M1: Attempt at correct method but makes a slip and has an extra − sign (and ends up with a quadratic with no real roots)
A0: Incorrect answer.

Part (c)
M1: At least one power decreasing in each component to give a vector.
A1: Correct answer.

Part (d)
M0: No method and incorrect answer.
A0: Follows M0.

Question 3 - Response C

(a) $(0^2-3(0)+7)i+(2(0)^2-3)j = 7i-3j$

speed $= |v|$ $\sqrt{7^2+(-3)^2} = 7.62\ ms^{-1}$

(b) $t^2-3t+7 = 2t^2-3$

$t^2+3t-10=0$

$(t-2)(t+5)=0$ $t=2$, $t=-5$

↳ reject the negative value

$t = 2$ seconds

(c) $a = v'$

$\frac{dv}{dt} = (2t-3)i+(4t)j = a$

(d) $2t-3 = -\frac{1}{4}t$

$\frac{9}{4}t = 3$ $t = \frac{4}{3}$ seconds

$t = 1.33$ seconds

7/ 9 marks

Q3 A B C

Part (a)
B1: Correct answer seen.
M1: Correct use of Pythagoras.
A1: Correct answer.

Part (b)
M1: Correct method equating **i** and **j** components of **v**.
A1: Correct answer.

Part (c)
M1: At least one power decreasing in each component to give a vector.
A1: Correct answer.

Part (d)
M0: A common incorrect method where they equate the **i** and **j** components of **a**.
A0: Follows M0.

Question 4

Question 4 - Introduction

This question tests a candidate's knowledge of constant acceleration in a vectors context, using the vector forms of the *suvat* formulae.

Question 4 - Question

4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})$ m s^{-2}

At time $t = 0$, P passes through the point A.

At time $t = 5$ s, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})$ m s^{-1}

(*a*) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})$ m.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})$ m.

(*b*) Find the value of T.

(3)

(*c*) Find the value of c.

(3)

(Total for Question 4 is 10 marks)

Question 4 - Mark Scheme

Question	Scheme	Marks	AOs
4(a)	$\mathbf{v}_B = (-16\mathbf{i} - 3\mathbf{j}) + 5(2.4\mathbf{i} + \mathbf{j})$	M1	3.4
	$\mathbf{v}_B = (-4\mathbf{i} + 2\mathbf{j})$	A1	1.1b
	$\sqrt{(-4)^2 + 2^2}$	M1	3.1a
	$\sqrt{20} =\ 2\sqrt{5}$, 4.5 or better ($\mathrm{m\ s^{-1}}$)	A1	1.1b
		(4)	
4(b)	<u>Using *A* as the initial position</u>: $\mathbf{r}_C = \mathbf{v}_A t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_A$ where $t = T$ $(4\mathbf{i} + c\mathbf{j}) = (-16\mathbf{i} - 3\mathbf{j})T + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})T^2 + (44\mathbf{i} - 10\mathbf{j})$ **OR** $\begin{pmatrix} 4 \\ c \end{pmatrix} = \begin{pmatrix} -16 \\ -3 \end{pmatrix} T + \frac{1}{2}\begin{pmatrix} 2.4 \\ 1 \end{pmatrix} T^2 + \begin{pmatrix} 44 \\ -10 \end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *T* only. Allow *t* instead of *T*. **N.B.** Allow omission of 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all i.e. $4 = -16T + \frac{1}{2} \times 2.4T^2$ scores M1A0A0	M1	3.1a
	$4 = -16T + \frac{1}{2} \times 2.4T^2 + 44$	A1	1.1b
	$(T =)\ 10$	A1	1.1b
	ALTERNATIVE <u>using *B* as the initial position</u>: (The position vector of *B*, $\mathbf{r}_B$, should be $-6\mathbf{i} - 12.5\mathbf{j}$ but no credit for finding this) $\mathbf{r}_C = \mathbf{v}_B t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_B$ using **their** $\mathbf{v}_B$ from (a) and **their** $\mathbf{r}_B$ $(4\mathbf{i} + c\mathbf{j}) = (-4\mathbf{i} + 2\mathbf{j})t + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})t^2 + (-6\mathbf{i} - 12.5\mathbf{j})$ $\begin{pmatrix} 4 \\ c \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 2.4 \\ 1 \end{pmatrix} t^2 + \begin{pmatrix} -6 \\ -12.5 \end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *t* only. Allow if they have *T* instead of *t*.	M1	3.1a

Q4

i ? A B C

	N.B. Allow omission of their -6 or if they use 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all. e.g. $4=-4t+\frac{1}{2}\times 2.4t^2$ scores M1A0A0		
	$4=-4t+\frac{1}{2}\times 2.4t^2-6$	A1	1.1b
	$t = 5$ so (T =) 10	A1	1.1b
		(3)	
4(c)	Equating **j**-components, with <u>their value of T or t substituted</u>, to give an equation, which must have a square term, in c only. **N.B.** Allow $\pm c$ in their equation. **(N.B.** Allow omission of -10 or their -12.5 for this M mark i.e. if using A as initial position $c=(-3\times 10)+\frac{1}{2}\times 1\times 10^2$ scores M1M0A0 **OR** if using B as initial position $c=(2\times 5)+\frac{1}{2}\times 1\times 5^2$ scores M1M0A0)	M1	2.1
	if using A as initial position $c=(-3\times 10)+\frac{1}{2}\times 1\times 10^2+(-10)$ **N.B.** Allow $\pm c$ and/or $\pm(-10)$ in their equation **OR** if using B as initial position $c=(2\times 5)+\frac{1}{2}\times 1\times 5^2+(-12.5)$ **N.B.** Allow $\pm c$ and/or $\pm(-12.5)$ in their equation	M1	1.1b
	c = 10	A1	1.1b
		(3)	
			(10 marks)

Notes: Accept column vectors throughout		
4a	M1	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 5$ to give an unsimplified $\mathbf{v}_B$ M0 if $\mathbf{u} = \mathbf{0}$ **N.B.** If using integration, they must get to the same stage i.e. have found the constant and put $t = 5$ M0 if they omit the constant altogether
	A1	Correct $\mathbf{v}_B$ with **i**'s and **j**'s collected
	M1	Use of Pythagoras on *their* $\mathbf{v}_B$ to give a magnitude (need the root)
	A1	Must be positive
4b	M1	Equating components of **i** to give an equation in T or t only. **N.B.** (they could use integration to get to the same stage) for this M mark, they **only** need to be equating the **i**-components, and receive no credit until they do so. M0 if $\mathbf{u} = \mathbf{0}$
	A1	A correct equation in T or t only (could be in $(T-5)$ if using B as initial position)
	A1	$T = 10$
4c	M1	Equating components of **j** to give an equation in c only but allow omission of their initial position
	M1	With their value of T or t and **must include $t = 0$ position** (should be -10 if using A **OR** their -12.5 if using B)
	A1	cao

Question 4 - Examiner Comments

The first part was usually well attempted with most candidates scoring all four marks. The vast majority used $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to find the velocity but a few lost the second two marks by not going on to find the magnitude of their velocity vector. Parts (b) and (c), however, proved to be significantly more challenging. A few candidates wrote down a correct vector equation but were unable to make any further progress without focussing on separate components. Those who used integration to find **r** were often more successful, because the initial position of A was the integration constant. Of those who used $\mathbf{r} = \mathbf{u}t + 0.5\mathbf{a}t^2$, many forgot to include the initial position of A and/or made a sign error and lost marks in both (b) and (c). In part (b), occasionally the quadratic equation was solved correctly but the wrong value of T was selected and in part (c) there was often a sign error on the 30 term. It was very rare to see candidates working with B as their initial position.

Q4

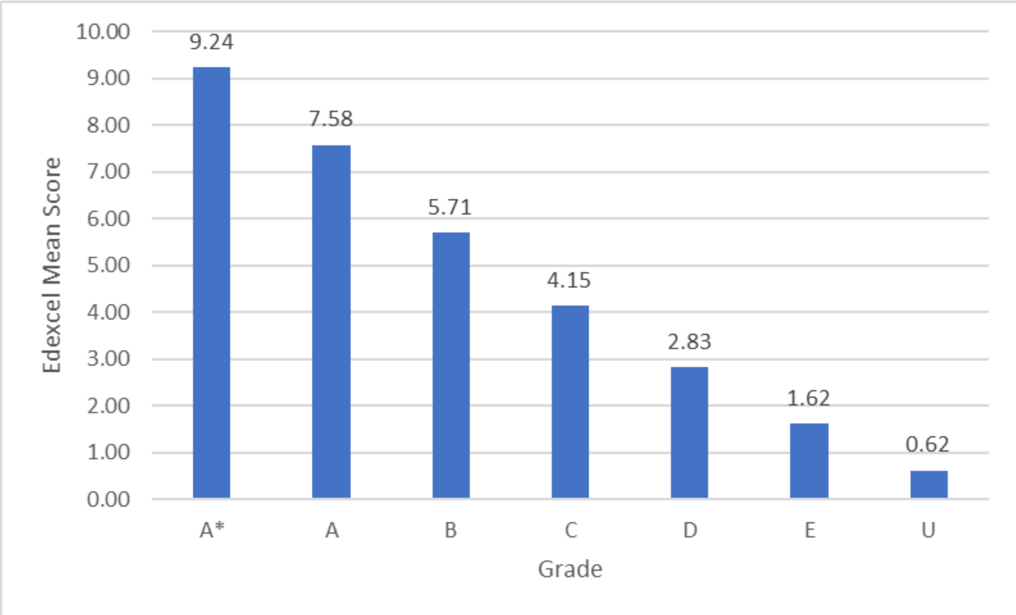
A

B

C

Question 4 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
5.63	10	56%	9.24	7.58	5.71	4.15	2.83	1.62	0.62



Question 4 - Response A

(a) $s=$

$u=(-16i-3j)ms^{-1}$

$V=?$

$a=(2.4i+j)ms^{-2}$

$t=5$

$$V=\begin{pmatrix}-16\\-3\end{pmatrix}+\begin{pmatrix}2.4\\1\end{pmatrix}\times 5$$

$$V=\begin{pmatrix}-16\\-3\end{pmatrix}+\begin{pmatrix}12\\5\end{pmatrix}$$

$$V=\begin{pmatrix}-4\\2\end{pmatrix}$$

$-4i+2j$

b) A $(44i-10j)$

C $t=Ts$ where $\cancel{T}>5$

C $(4i+cj)$

$$s=\begin{pmatrix}4-44\\c--10\end{pmatrix}=\begin{pmatrix}-40\\c+10\end{pmatrix}$$

$u=(-16i-3j)$

$V=$ $\qquad s=ut+\frac{1}{2}at^2$

$a=(2.4i+j)$

$T=?$

$$\begin{pmatrix}-40\\c+10\end{pmatrix}=\begin{pmatrix}-16\\-3\end{pmatrix}t+\frac{1}{2}\begin{pmatrix}2.4\\1\end{pmatrix}t^2$$

$$\begin{pmatrix}1.2\\0.5\end{pmatrix}t^2+\begin{pmatrix}-16\\-3\end{pmatrix}t-\begin{pmatrix}-40\\c+10\end{pmatrix}=0$$

2/10 marks

Part (a)
M1: Uses $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with t = 5, to obtain an unsimplified $\mathbf{v}_B$.
A1: Correct velocity.
M0: Does not use Pythagoras on their $\mathbf{v}_B$
A0: Following M0.

Part (b)
M0: Does not equate the **i**-components to obtain a quadratic equation in T only.
A0: Following M0.
A0: Following M0.

Part (c)
M0: Does not equate the **j**-components with their value of T substituted to obtain an equation in c only.
M0: Does not equate the **j**-components with their value of T substituted to obtain an equation in c only which includes the position at $t = 0$.
A0: Following M0.

Examiner comments
There was no credit available in parts (b) and (c) for vector equation(s) and many candidates faltered at this point and were unable to make progress.

Q4

A
B
C

Question 4 - Response B

(c) Find the value of c. $\rightarrow$ 2.4i + j **(3)**

i component A B

a) s:
u: -16 v = u + at
v: v = -16 + 2.4×5
a: 2.4 = -4 i
t: 5

j component
s: v = u + at
u: -3 v = -3 + 1×5
v: v = 2j
a: 1
t: 5 speed through B = -4i + 2j

Question 4 continued

b) in i direction

$s = -40$

$u = -16$ $\quad$ $s = ut + \frac{1}{2}at^2$

$v =$ $\quad$ $-4 = -16T + \frac{1}{2} \times 2.4T^2$

$a = 2.4$ $\quad$ $1.2T^2 - 16T + 4 = 0$

$\boxed{t = T}$ $\quad$ $T = 13.078$

$T = 0.255$

~~Bn~~ $T > 5 \therefore T = 13.078$

c) in j direction

$\boxed{s = c}$

$u = -3$ $\quad$ $s = ut + \frac{1}{2}at^2$

$v =$ $\quad$ $c = -3(13.078) + \frac{1}{2} \times 1 \times 13.078^2$

$a = 1$ $\quad$ $c = 46.3$

$t = 13.078$

$c = 46.3$

4/10 marks

Part (a)
M1: Uses **v** = **u** + **a**t with t = 5 to give an unsimplified $\mathbf{v}_B$
A1: Correct $\mathbf{v}_B$
M0: No attempt to find the magnitude using Pythagoras
A0: Follows M0.

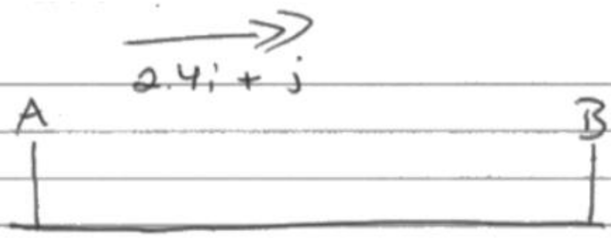
Part (b)
M1: Equates **i** components of position vector, using A as the initial position, to give a quadratic in *T*. (Note that a correct displacement of −40 is stated but there is a slip when substituting into equation)
A0: Incorrect equation.
A0: Follows A0

Part (c)
M1: Equates j components with their value of *T* substituted to give equation in c only (using A as initial position) condone omission of initial position.
M0: Omits to include initial position term.
A0: Follows M0

Question 4 - Response C

(3)

4a)



$S = x$

$u = (-16i - 3j)$

$v = ?$

$a = (2.4i + j)$

$t = 5$

Speed $= \sqrt{4^2 + 2^2} = \sqrt{20}$

$= \underline{4.47}$

$V = (-16i - 3j) + (12i + 5j)$

$V = (-4i + 2j)$

4b) ~~44i − 4 = 40i~~
~~−10j − cj = −10 − cj~~

~~40i − 10 − cj =~~

$S = (-16ti - 3tj) + \frac{1}{2}(2.4i + j)t^2$

$S = -40i, -10 - cj$

$-40i = -16ti + 1.2t^2$

$1.2t^2 - 16t + 40 = 0$

$\cancel{t = 10}$ $\quad t = \frac{10}{3}$ $\quad T = \frac{10}{3} = \underline{3.34 \text{ sec}}$

4c) $c = -3\left(\frac{10}{3}\right) + 0.5(1)\left(\frac{10}{3}\right)^2$

$c = -\frac{40}{9} = \underline{-4.44}$

7/ 10 marks

Part (a)
M1: Uses **v** = **u** + **a**t, with t = 5, to obtain an unsimplified $\mathbf{v}_B$
A1: Correct velocity.
M1: Uses Pythagoras on their $\mathbf{v}_B$
A1: Correct speed.

Part (b)
M1: Equates **i**-components to obtain a quadratic equation in *T* only, using A as initial position.
A1: Correct equation.
A0: Incorrect answer for *T* as $T > 5$

Part (c)
M1: Equates **j**-components with their value of *T* substituted to obtain an equation in c only, using A as initial position, condone omission of -10
M0: Incorrect method as -10 has been omitted.
A0: Incorrect answer for c

Question 5

Introduction | Question | Mark Scheme | Examiner Comments
Performance | Response A | Response B | Response C

Question 5 - Introduction

This question tests a candidate's knowledge of projectile motion.

Question 5 - Question

5.

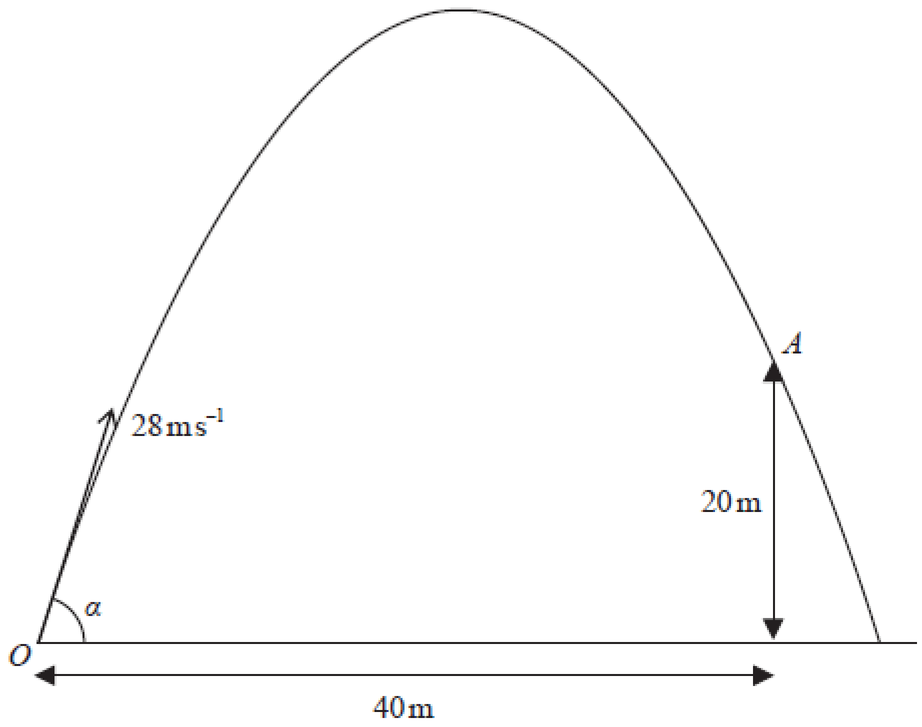


Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to (*a*) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(*b*) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(*c*) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

The model does not include air resistance.

(*d*) State one other limitation of the model.

(1)

(Total for Question 5 is 11 marks)

Question 5 - Mark Scheme

Question	Scheme	Marks	AOs
	N.B. In this question, allow misread of α for *a*.		
5(a)	Use horizontal motion to give an equation in T and α only: $28\cos\alpha \times T = 40$	M1	3.4
	$T = \dfrac{10}{7\cos\alpha}$ *	A1*	1.1b
		(2)	
5(b)	Use vertical motion to give an equation in T and α only	M1	3.3
	$20 = (28\sin\alpha)T - \frac{1}{2}gT^2$	A1	1.1b
	Eliminate T to give an unsimplified equation in α only: $20 = (28\sin\alpha)\times\dfrac{10}{7\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{10}{7\cos\alpha}\right)^2$	M1	1.1b
	Use $\sec^2\alpha = 1+\tan^2\alpha$ oe to give an unsimplified equation in **tan** α **only** : $20 = 40\tan\alpha - \dfrac{1}{2}g\times\dfrac{100}{49}(1+\tan^2\alpha)$	M1	3.1b
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$ * (allow $0 = \tan^2\alpha - 4\tan\alpha + 3$)	A1*	2.2a
		(5)	
5(c)	Solve and use of $\tan\alpha = 3$ or $\sin\alpha = \dfrac{3}{\sqrt{10}}$ or $\alpha = 71.565..^{o}$ to find an equation in H only.	M1	3.1b
	$0 = (28\sin\alpha)^2 - 2gH$ **where** $\tan\alpha = 3$ $(\alpha = 71.565..^{o})$	M1	3.4
	H = 36 or 36.0 (m)	A1	1.1b
		(3)	
5(d)	e.g. spin of the ball, the wind, the dimensions or shape of the ball, ball is modelled as a particle, uses an inaccurate value of *g*, motion takes place in 3D not in 2D, *g* could be variable. B0 if mass or weight are mentioned. B0 for ground may not be horizontal.	B1	3.5b
		(1)	
			(11 marks)

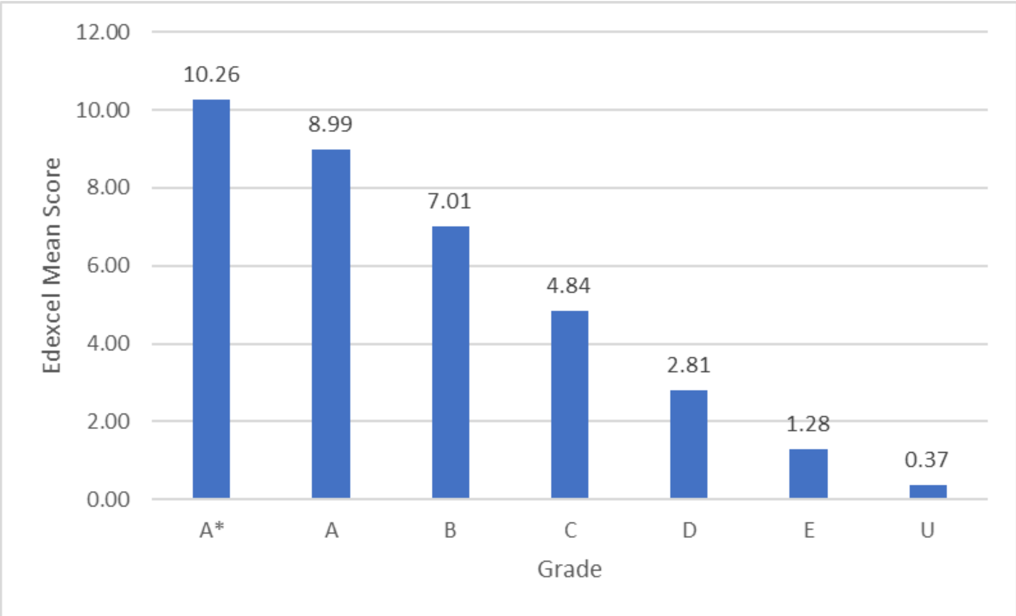
Notes:		
5a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1*	Correct printed answer correctly obtained. Allow $\frac{10}{7\cos\alpha} = T$ OR $T = \frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha}$ OR $\frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha} = T$ OR t instead of T
5b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Eliminate T, using either the given answer in (a) or their own T expression, from their equation to give an unsimplified equation in α only
	M1	Use $\sec^2\alpha = 1 + \tan^2\alpha$ to produce an equation in $\tan\alpha$ only
	A1*	Given answer correctly obtained. **N.B.** Must be α (or a) in the final answer but allow a different angle in the working.
5c	M1	Solve given equation and select larger value of $\tan\alpha$ and use it to try to obtain an equation in H only.
	M1	Complete method to give an equation in ***H* only**, using **<u>larger</u>** value of α, correct no. of terms, dim correct, condone sin/cos confusion and sign errors.
	A1	cao. Must be positive, (allow a negative value, changed to a positive answer). **N.B.** This answer comes from use of $g = 9.8$, so must be rounded to 2 or 3 sf.
5d	B1	B0 if any incorrect extras

Question 5 - Examiner Comments

In part (a), the vast majority of candidates recognised that the horizontal component of velocity of the particle remained constant throughout the motion; this was then used, together with the horizontal distance, to form an equation in T and α and to derive the given result. The second part, however, provided a greater challenge and, although many realised that they needed to consider the vertical motion, a few did not know how to proceed, with some solving the given quadratic in tanα rather deriving it. However, many did make progress, using $s = ut + \frac{1}{2}at^2$ with a vertical distance of 20 although there were occasional sign errors. They mostly then substituted the expression for T from part (a) to obtain an equation in α although slips were not uncommon at this stage. Those starting with different *suvat* equations tended to make little significant progress. To achieve the given result it was necessary to use the trig identity $\sec^2\alpha = 1 + \tan^2\alpha$ (or equivalent work) to obtain an unsimplified quadratic equation in $\tan\alpha$ before reaching the given equation. This was the main stumbling block for many and it was not unusual to see candidates make more than one attempt. Since the final equation was given and the demand of the question was "show that", it was essential that all steps in the working were correct and the final answer was stated exactly as printed in order to achieve all the marks. In part (c) most candidates realised they should solve the given quadratic in tanα to give two possible angles of projection. Consideration of the situation should have indicated that the larger of the two angles would lead to the greatest possible height. Although many recognised that the greatest height is reached when the vertical component of velocity is zero, some chose to use the smaller 45° angle. Some found the greatest heights for both angles but then failed to select the relevant one. Other errors included using the time to greatest height as half the answer in part (a) or choosing $\sin\alpha = 1$ to give the maximum height in an otherwise correct *suvat* equation. Rounding errors were not uncommon leaving the final answer incorrect to 3 significant figures. In the final part, most candidates were able to suggest an acceptable limitation of the model such as it not including spin, dimensions of the ball, effect of wind or a more accurate value for g. The most common incorrect answers were that the model did not include weight/mass of the ball and the ground was not perfectly flat which was not specified as a modelling assumption in the question.

Question 5 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
6.46	11	59%	10.26	8.99	7.01	4.84	2.81	1.28	0.37



Question 5 - Response A

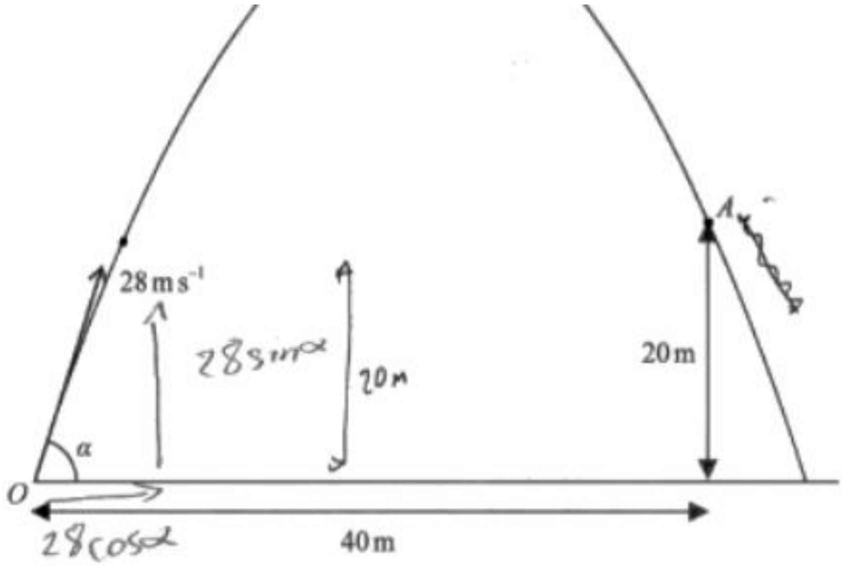


Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$ **(2)**

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$ **(5)**

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A. **(3)**

The model does not include air resistance.

(d) State one other limitation of the model. **(1)**

5a. $R \rightarrow$ $s = ut$

$40 = 28\cos\alpha \, T \Rightarrow T = \dfrac{40}{28\cos\alpha} = \dfrac{10}{7\cos\alpha}$ (1)

Question 5 continued

$R\uparrow$ $s = 20$

$u = 28\sin\alpha$ $\qquad s = ut + \frac{1}{2}at^2$

$v =$

$a = -9.8$ $\qquad 20 = 28\sin\alpha + \frac{1}{2}(-9.8)\,T^2$ ②

$t = T$

sub ① into ②

$20 = 28\sin\alpha - 4.9\left(\frac{10}{7\cos\alpha}\right)^2$

$20 = 28\sin\alpha - \frac{490}{49\cos^2\alpha}$

$980 \quad 20\cos^2\alpha = 28\sin\alpha + \frac{490}{49}$ $\qquad 20\cos^2\alpha = 28\sin\alpha + 100$

$R\uparrow$ $s = 20$ $\qquad v^2 = u^2 + 2as$

$u = 28\sin\alpha$ $\qquad v^2 = (28^2\sin^2\alpha) + 2(-9.81)(20)$

$v = ?$ $\qquad v^2 = 784\sin^2\alpha - 392.4$

$a = -9.81$

$t =$

$R\uparrow$ $s =$ $\qquad v = u + at$

$u = 28\sin\alpha$ $\qquad V = 28\sin\alpha + (-9.81)\,T$

$v:$

$a = -9.81$

$t = T$

$R\uparrow$ $s = 20$

$u = \sqrt{784\sin^2\alpha - 392.4}$

$v =$ $\qquad \cancel{20 = T\sqrt{784\sin^2\alpha}}$

$a = -9.81$

$t = T$ $\qquad 20\cos^2\alpha - 28\sin\alpha - 100 = 0$

d. The model does not consider the spin of the ball

5/11 marks

Part (a)

M1: Uses horizontal motion to obtain a dimensionally correct equation with the correct number of terms, in T and α only,

A1*: Correct *printed* answer, correctly obtained.

Q5 i ? A B C

Part (b)

There are 2 attempts at (b) but the first seems the most complete so we mark that one.

M1: The candidate quotes a correct suvat formula and then uses the vertical motion to attempt to obtain a dimensionally correct equation with the correct number of terms, in T and α only. There is then a slip, with 't' being omitted. The M1 is still awarded because the formula has been stated first so this omission is considered to be a slip.
N.B. If the formula had not been stated, this would be M0.

A0: Incorrect equation.

M1: Eliminates T correctly for their equation to give an unsimplified equation in α only.

M0: Does not use $\sec^2\alpha = 1 + \tan^2\alpha$ to obtain an equation in tanα only

A0*: Follows M0

Part (c)

M0: Does not solve the given equation and so does not find the larger value of α and use it.

M0: Incomplete method, as no larger value of α obtained and used to obtain an equation in H (s) only.

A0: Follows M0

Part (d)

B1: This is an appropriate limitation of the model.

Question 5 - Response B

a) $s = ut + \frac{1}{2}at^2$

$s_x = u_x t + \frac{1}{2} a_x t^2$

$40 = 28 \cos\alpha \, T$

$T = \dfrac{40}{28\cos\alpha} = \dfrac{10}{7\cos\alpha}$

b) $\tan^2\alpha - 4\tan\alpha + 3 = 0$

$\dfrac{\sin^2\alpha}{\cos^2\alpha} - \dfrac{\sin\alpha}{\cos\alpha} + 3 = 0$

$s_y = u_y t + \frac{1}{2} a_y t^2$

$20 = 28\sin\alpha \, T - 4.9T^2$

$20 = \dfrac{28\sin\alpha \times 7\cos\alpha}{10} - \dfrac{4.9 \times 10^2}{49\cos^2\alpha}$

$20 = \dfrac{98\sin\alpha\cos\alpha}{5} - \dfrac{10}{\cos^2\alpha}$

$100\cos^2\alpha = 98\sin\alpha\cos^3\alpha - 50$

$50\cos^2\alpha = 49\sin\alpha\cos^3\alpha - 25$

Q5

A

B

C

Question 5 continued b)

~~y~~

$s = ut + \frac{1}{2}at^2$

$s = 20$ at $t = \frac{10}{7\cos\alpha}$

$20 = 28\sin\alpha \times \frac{10}{7\cos\alpha} + \frac{1}{2}(-9.8)\left(\frac{100}{49\cos^2\alpha}\right)$

$\cancel{20 = 280\frac{\sin\alpha}{7\cos\alpha}}$

$20 = 40\frac{\sin\alpha}{\cos\alpha} - \frac{10}{\cos^2\alpha}$

$\cancel{20 = 40\frac{\sin\alpha}{\cos\alpha} - \frac{10}{1-\sin^2\alpha}}$

$\sin^2\alpha + \cos^2\alpha = 1$

$10 = 10\sin^2\alpha + 10\cos^2\alpha$

$20 = 40\tan\alpha - \frac{10\sin^2\alpha + 10\cos^2\alpha}{\cos^2\alpha}$

$20 = 40\tan\alpha - 10\tan^2\alpha - 10$

$4\tan\alpha - \tan^2\alpha - 3 = 0$

$\tan^2\alpha - 4\tan\alpha + 3 = 0$

c) $s_y = 28t\sin\alpha - 4.9t^2$

$\tan\alpha = 1, 3$

$\alpha = 45°, 71.6°$

$T = \frac{10}{7\cos\alpha}$

$s_x = ut$
$40 = 28\cancel{T}\cos\alpha$

$\cancel{x}$

$\frac{40+x}{2} = 28T_2\cos\alpha \rightarrow T_2 = \frac{40+x}{56\cos\alpha}$

$s_{y\,max} = 28T_2\sin\alpha - 4.9T_2^2$
$= 28\sin\alpha\left(\frac{40+x}{56\cos\alpha}\right) - 4.9\left(\frac{40+x}{56\cos\alpha}\right)^2$

$40 + x = 28\cos\alpha \times T_{full}$

$0 = 28\sin\alpha\, T_{full} - 4.9\, T_{full}^2$

$T_{full} = \frac{28\sin\alpha}{4.9}$

$\frac{1}{2}T_{full} = T$ at $s_{y\,max} = \frac{14\sin\alpha}{4.9}$

$s_{y\,max} =$

7/11 marks

Part (a)
M1: Uses horizontal motion to obtain a dimensionally correct equation with the correct number of terms, in T and α only,
A1*: Correct *printed* answer, correctly obtained.

Part (b)
M1: Uses vertical motion to obtain a dimensionally correct equation with the correct number of terms, in T and α only,
A1: Correct equation.
M1: Eliminates T to give an unsimplified equation in α only.
M1: Derives and uses $\sec^2\alpha = 1 + \tan^2\alpha$ to obtain an equation in $\tan\alpha$ only
A1*: Correct *printed* answer, correctly obtained.

Part (c)
M0: Solves the given equation but never attempts to use the larger value of α to obtain an equation in H.
M0: No method, using the larger value of α to obtain an equation in H (their s) only.
A0: After M0.

Part (d)
B0: No attempt.

Question 5 - Response C

Question 5 continued

a) ↔ : ~~28cos α~~ $v = 28\cos\alpha$, $s = x$

$v = \frac{s}{t}$, at A: $28\cos\alpha = \frac{40}{T}$

$\therefore T = \frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha}$

b) ↕ : $s =$ ~~20~~ ~~α~~ y

$u = 28\sin\alpha$

$v = ?$

$a = -g$

$t = ?$

at A: $s = 20$, $t = T$

"$s = ut + \frac{1}{2}at^2$" → $20 = 28\sin\alpha \cdot T + \frac{1}{2}(-g)T^2$

$20 = 28\sin\alpha\left(\frac{10}{7\cos\alpha}\right) - \frac{g}{2}\left(\frac{100}{49\cos^2\alpha}\right)$

$20 = 40\tan\alpha - 10\sec^2\alpha$

~~40~~ $-10\sec^2\alpha + 40\tan\alpha - 20 = 0$, $\sec^2\alpha \equiv 1 + \tan^2\alpha$

$(1+\tan^2\alpha) - 4\tan\alpha + 2 = 0$ $\therefore$ $\tan^2\alpha - 4\tan\alpha + 3 = 0$

c) S_{max} occurs when $x = 20$, $t = t_s$

$\therefore t_s$ ~~x~~ $= \frac{20}{28\cos\alpha} = \frac{5}{7\cos\alpha} = \frac{5}{7}\sec\alpha$

$S_{max} = ut_s + \frac{1}{2}a t_s^2$

~~tan α~~ $\tan\alpha = 3, 1$ $\therefore \alpha = 45, 71.6...$

$\therefore t_s = 2.29, 1.01...$

$t_s = 2.29$ when $\alpha = 71.6$

Q5

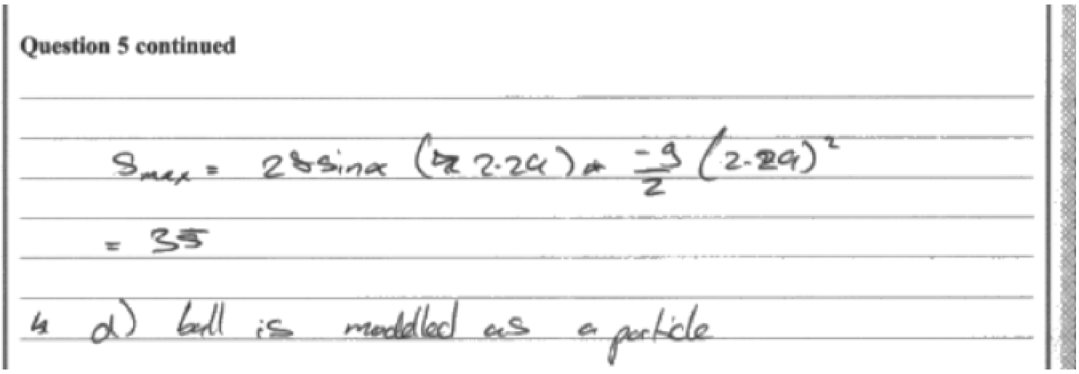
i

?

A

B

C



Question 5 continued

$S_{max} = 28\sin\alpha\,(t = 2.24) + \frac{-g}{2}(2.24)^2$

$= 35$

4 d) ball is modelled as a particle

9/11 marks

Part (a)

M1: Uses horizontal motion to obtain a dimensionally correct equation with the correct number of terms, in T and α only.

A1*: Correct *printed* answer, correctly obtained.

Part (b)

M1: Uses vertical motion to obtain a dimensionally correct equation with the correct number of terms, in T and α only.

A1: Correct equation.

M1: Eliminates T to give an unsimplified equation in α only.

M1: Uses $\sec^2\alpha = 1 + \tan^2\alpha$ to obtain an equation in $\tan\alpha$ only.

A1*: Correct *printed* answer, correctly obtained.

Part (c)

M1: Solves the given equation and uses the larger value of α to obtain an equation in H (s) only.

M0: Does use the larger value of α but incorrect method to find an equation in H only, since uses half the x value to A ($x = 20$) to find the time to the top.

A0: Following M0.

Part (d)

B1: A correct limitation.

Question 6

Introduction | Question | Mark Scheme | Examiner Comments
Performance | Response A | Response B | Response C

Question 6 - Introduction

This question tests a candidate's knowledge of statics of rigid bodies.

Question 6 - Question

6.

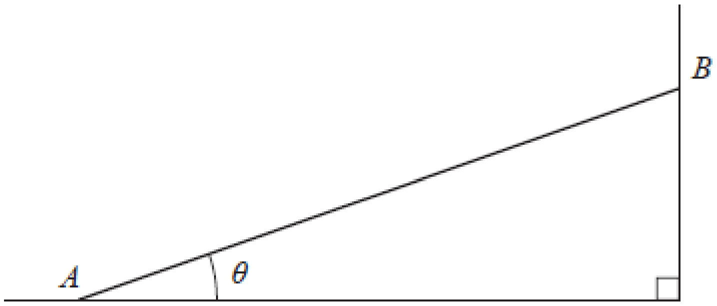


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(*a*) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (*b*), (*c*) and (*d*).

(*b*) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \dfrac{3}{4}$

(*c*) find the value of μ

(5)

(*d*) find, in terms of *M* and *g*, the magnitude of the resultant force acting on the rod at *A*.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to *B* than it is to *A*.

A new value for *S* is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(*e*) State whether this new value for *S* is larger, smaller or equal to the value that *S* would take using the initial model. **Give a reason for your answer.**

(1)

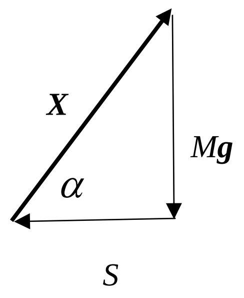
(Total for Question 6 is 13 marks)

Question 6 - Mark Scheme

Question	Scheme	Marks	AOs
6(a)	The normal reaction at *B* is acting to the left so it must act to the right, right as it needs to balance (oppose, counter) the force at *B*, right as it prevents the rod from sliding (slipping, falling), right as the weight (mass) of the rod will mean the rod tends to slip left, mass or weight will be pushing the rod to the left so friction will oppose that. **N.B.** You may see an arrow on the diagram at *A*, instead of 'right'. B0 if they say the rod is moving oe Accept towards the wall instead of to the right.	B1	2.4
		(1)	
6(b)	Take moments about *A*	M1	3.4
	$S\times 2a\sin\theta = Mga\cos\theta$	A1	1.1b
	$S = \frac{1}{2}Mg\cot\theta$ *	A1*	2.2a
		(3)	
6(c)	Resolve vertically, $R = Mg$	B1	3.3
	Resolve horizontally, $F = S$	B1	3.3
	Other possible equations: Resolve along the rod, $F\cos\theta + R\sin\theta = S\cos\theta + Mg\sin\theta$ Resolve perp to the rod, $R\cos\theta + S\sin\theta = F\sin\theta + Mg\cos\theta$ M(*B*), $R\times 2a\cos\theta = F\times 2a\sin\theta + Mga\cos\theta$ M(*G*), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$		
	$F = \mu R$	B1	1.2

	$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$	**dM**1	2.1
	$\mu = \frac{2}{3}$ oe Accept 0.67 or better	A1	2.2a
	S.C. For $F \leqslant \mu R$, B0 $\frac{1}{2}Mg \times \frac{4}{3} \leqslant \mu Mg$ M1 $\frac{2}{3} \leqslant \mu$ A0 **N.B.** If $\mu = \frac{2}{3}$ follows this, they could score all the marks.		
		(5)	
6(d)	$\sqrt{F^2 + R^2}$	M1	3.1a
	$\sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$	M1	1.1b
	$\frac{1}{3}Mg\sqrt{13}$ or 1.2Mg or better	A1	2.2a
		(3)	
6(e)	New value of S would be **larger** as the **moment** of the **weight** about $\boldsymbol{A}$ would be larger	B1	3.5a
		(1)	
			(13 marks)

Notes:		
6a	B1	Any equivalent appropriate statement.
6b	M1	Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors. **N.B.** If a's never appear, M0
	A1	Correct equation
	A1*	Correct given answer correctly obtained, **with no wrong working seen.** Allow $\frac{1}{2}Mg\cot\theta = S$ or $S = \frac{Mg\cot\theta}{2}$ or $\frac{Mg\cot\theta}{2} = S$ or $S = \frac{Mg}{2}\cot\theta$ or similar but NOT $S = \frac{1}{2}\cot\theta$ Mg or similar **N.B.** Allow m instead of M Must be θ in final answer but allow a different angle in the working.
6c	B1	cao
	B1	cao
	B1	Seen anywhere, e.g. on the diagram
	dM1	Using $F = \mu R$, their two equations and substitute for trig (not necessarily correctly) to produce an equation in μ only.

		This mark is **dependent** on the 3 previous B marks.
	A1	Accept 0.67 or better
6d	M1	Use of Pythagoras with square root to find the required magnitude, but F and R do not need to be substituted
	M1	Substitute for their F and their R in terms of Mg and take square root to obtain magnitude in terms of M and g only. **N.B.** Must be using Pythagoras
		ALTERNATIVE: Using trig on triangle of forces  M1: $X = \dfrac{Mg}{\sin\alpha}$ or $\dfrac{S}{\cos\alpha}$ M1: substitute for $\sin\alpha$ or $\cos\alpha$ and S, where $\tan\alpha = \dfrac{Mg}{S}$ $\left(= \dfrac{3}{2}\right)$, to obtain X in terms of M and g only.
	A1	Any equivalent surd form or $1.2Mg$ or better Must be in terms of M and g
6e	B1	Correct answer and any equivalent appropriate statement.

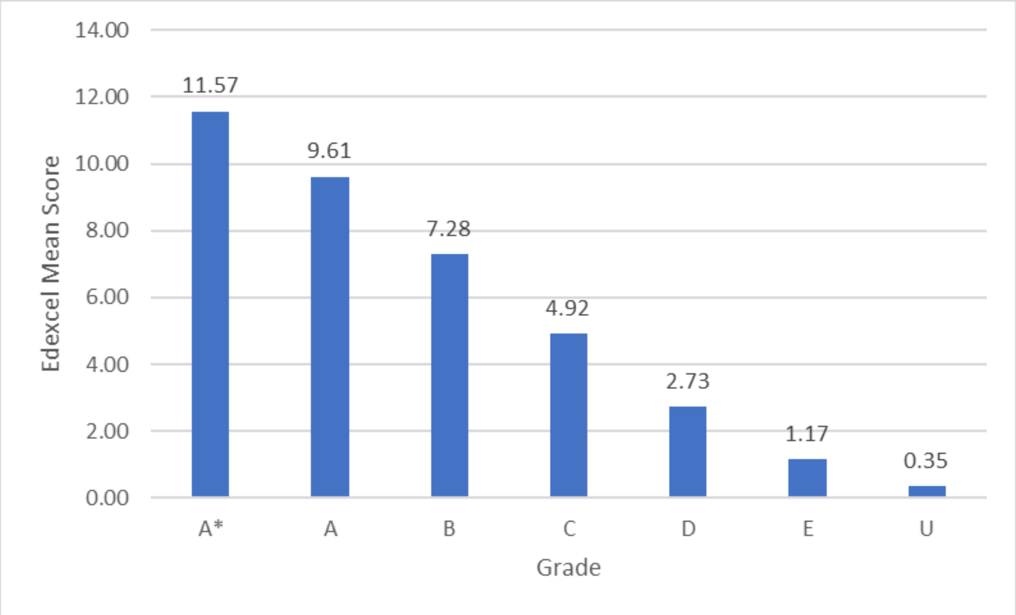
Question 6 - Examiner Comments

In the first part, the majority of candidates stated that the frictional force would act towards the right on the diagram and many gave a satisfactory reason. A minority gave the reason that the friction had to balance the reaction force at the wall which acted to the left. Most mentioned that it prevented the rod from slipping to the left. Those who did not achieve the mark often used language that implied the rod was actually moving. A small number thought that the direction was left or gave no reason at all. In part (b), the question directed the candidates to take moments about A and most were able to do so successfully with many achieving the three available marks. The occasional incorrect response had the distance to the reaction at B unresolved as $2a$. Also a small number had a's missing from both sides of their equation. Since this was a "show that" question it was important that all stages of the solution were shown and were correct. It should be noted that the candidate's final answer had to be written exactly as the printed answer, using the same symbols in the same order, to achieve the final mark. Part (c) required a calculation of the coefficient of friction. There were many fully correct solutions involving vertical and horizontal resolution but many incorrect responses were also seen. The majority who tried other approaches, such as moments about B or resolving parallel and perpendicular to the rod, were usually unsuccessful as they would often miss out a term or there would be a sin/cos error. Some candidates attempted to resolve vertically and horizontally but included an extra force. It was not uncommon for the horizontal resolution $F = S$ to be correct but an extra force would appear in the vertical resolution. Almost all recalled $F = \mu R$ and tried to use it in some way. Part (d) involved finding the magnitude of the resultant force acting at A. It was not always well answered and sometimes omitted entirely. A significant minority of responses were, however, fully correct and there were many who, having made an error in part (c), were still able to apply Pythagoras' theorem correctly with their values of F and S. However, some candidates did not understand what was required with $R = mg$, or an attempt to just combine R and F by adding, seen quite often. A small number failed to deal properly with the square root making an error with the simplification in terms of mg in an otherwise correct solution. In the final part, the majority of candidates were unable to gain the mark often because of lack of precision in the wording of their response. Many stated that S was larger because the weight was closer to B or it exerted more force/pressure at B. Some who did refer to an increased moment about A failed to specify it as the moment of the weight. Some mentioned distance but did not explicitly refer to moments. Candidates who used their moments equation to show that a distance of the weight from A greater than a led to a larger value of S were generally successful. A small number thought S was smaller or did not change.

Q6

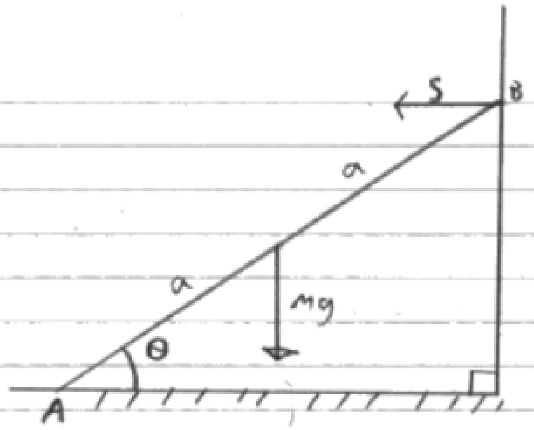
Question 6 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
6.89	13	53%	11.57	9.61	7.28	4.92	2.73	1.17	0.35



Question 6 - Response A

Question 6 continued

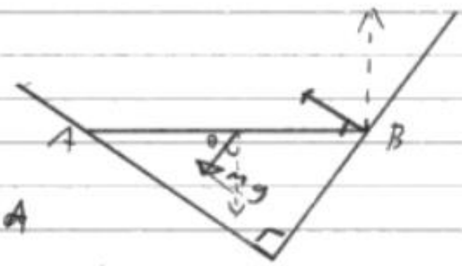


a) right as ~~A is on a rough horizontal ground~~ ~~A~~ if A was ~~On a~~ on ~~a~~ smooth ~~vertical~~ horizontal ground A would 'slip' towards the left ~~to~~ as A is On rough ground the frictional force must oppose ~~this~~ the tendency to 'slip'.

b) $\downarrow = \uparrow$ $\rightarrow = \leftarrow$

$R_A = Mg$ $F_r = S$

taking moments about A



$a \cdot mg\cos(\theta) = 2a \cdot S$

$\frac{1}{2}mg$

~~moment~~

$mg\cos(\theta) \times a = 2a \cdot S$

$a\,mg\cos(\theta) = 2aS$

$mg\cos(\theta) = 2S$

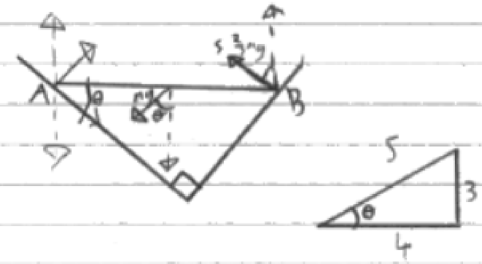
$S = \frac{1}{2}Mg\cos\theta$

$S = \frac{1}{2}Mg\cos\theta$

c) $F_{max} = \mu R_A$

$\downarrow = \uparrow$ $\quad$ $\rightarrow = \leftarrow$

$\cancel{R} = Mg = R_A$ $\quad$ $S = Fr$



$\tan(\theta) = \frac{3}{4}$

$\cos\theta = \frac{4}{5}$

Moments about A

$S = \frac{1}{2} Mg\cos\theta$

$\cos\theta = \frac{4}{5}$

$S = \frac{2}{5} Mg$

e) Smaller as your dividing by a larger number

4/13 marks

Part (a)
B1: Correct answer and acceptable reason.

Part (b)
M0: Attempts to take moments about A but does not use perpendicular distance for S (sin theta missing).
A0: Follows M0
A0*: Follows M0

Part (c)
B1: Correct vertical resolution
B1: Correct horizontal resolution.
B1: $F=\mu R$ seen
dM0: Does not use the equations and substitute for trig to produce an equation in mu only
A0: After dM0.

Part (d)
M0: No attempt to find the magnitude.
M0: Not available.
A0: After M0.

Q6

A
B
C

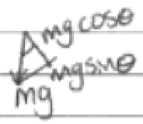
Part (e)
B0: Incorrect answer

Question 6 - Response B

a right because the mass of the rod will be pushing it left and friction is opposite to that force

b ~~M(A) cw a mg~~

mg



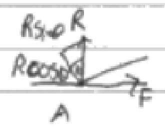
	cw		acw
M(A)	$amg\cos\theta$	$=$	$2aS\sin\theta$
$\div a$	$mg\cos\theta$	$=$	$2S\sin\theta$
$\div \sin$	$mg\cot\theta$	$=$	$2S$
$\div 2$	$\frac{1}{2}mg\cot\theta$	$=$	S

$\therefore S = \frac{1}{2}mg\cot\theta$

c $\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$



M(B)	cw		acw
	$2aR\cos\theta$	$=$	$amg\cos\theta$
	$2R$	$=$	mg

$F = S$

$F = \mu R \quad \therefore S = \mu R$

c cont.

$S\ F = \mu R$

$\mu R = \frac{1}{2} mg \cot\theta$

$2R = mg$

$\mu R = \frac{1}{2}(2R)\cot\theta$

$\mu = (\frac{1}{2} \times 2) \cot\theta$

$\mu = \cot\theta$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\mu = \frac{4}{3}$

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

$\frac{4}{5} \div \frac{3}{5} = \frac{4}{3}$

d

e larger as it would be pushing into the wall harder and ∴ the reaction greater than before

6/ 13 marks

Part (a)
B1: Correct answer and acceptable reason.

Part (b)
M1: Takes moments about A, dimensionally correct with correct terms.
A1: Correct equation.
A1*: Correct printed answer, correctly obtained.

Part (c)
B0: Moments about B equation has a term missing.
B1: Correct horizontal resolution.
B1: $F = \mu R$ seen
dM0: Dependent on previous 3 B marks.
A0: After dM0.

Part (d)
M0: No attempt to find the magnitude.
M0: Not available.
A0: After M0.

Part (e)
B0: Correct answer but insufficient explanation, no mention of moment of weight about A.

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Question 6 - Response C

6.

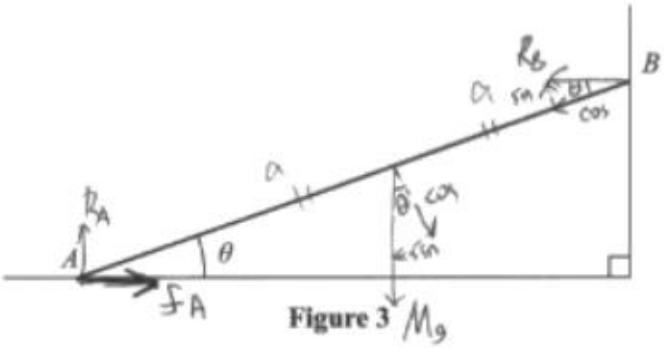


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.** let friction of A is fA

the ~~force~~ rough surface of ground due to the rode

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2} Mg \cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

Q6

A

B

C

Question 6 continued

b) At A

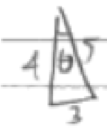
$\cos\theta \, Mg \cdot a = \sin\theta \, S \cdot 2a$

$S = \dfrac{\cos\theta \, Mga}{\sin\theta \, 2a} = \frac{1}{2}\cot\theta \, Mg$

c)

$\perp$: $R_A = Mg$

$//$: $R_B = f_A = S = \mu Mg$



$S = \frac{1}{2}Mg \dfrac{\cos\theta}{\sin\theta} = \frac{1}{2} Mg \dfrac{\frac{4}{5}}{\frac{3}{5}} = \frac{2}{3}Mg$

so $\frac{2}{3}Mg = \mu Mg$

$\mu = \frac{2}{3}$

d) $R_A = Mg$, $f_A = \frac{2}{3}Mg$

so, $F_A = \sqrt{R_A^2 + f_A^2} = \sqrt{(Mg)^2 + (\frac{2}{3}Mg)^2} = \frac{\sqrt{13}}{3}Mg$

e) ~~not change~~ S will be larger, larger distance from A to centre mass <

10/13 marks

Part (a)
B0: Correct answer given as an arrow on the diagram at A, but no acceptable reason.

Part (b)
M1: Takes moments about A, dimensionally correct with correct terms.
A1: Correct equation.
A0*: This form is not allowed.

Q6

A
B
C

Part (c)

B1: Correct vertical resolution.
B1: Correct horizontal resolution.
B1: F = μR seen
dM1: Dependent on previous 3 B marks. Uses F = μR, their two equations and substitutes for trigonometry to produce an equation in μ only.
A1: Correct value.

Part (d)

M1: Uses Pythagoras, with square root to find the magnitude of the resultant force.
M1: F and R have been substituted, and square root taken to obtain magnitude in terms of M and g only.
A1: Correct magnitude

Part (e)

B0: Correct answer but insufficient explanation as no mention of the moment of the weight about A.